

Does $f(R, T)$ gravity admit a stationary scenario between dark energy and dark matter in its framework?

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Abstract

In this note we address the well-known cosmic coincidence problem in the framework of the $f(R, T)$ gravity. In order to achieve this, an interaction between dark energy and dark matter is considered. A constraint equation is obtained which filters the $f(R, T)$ models that produce a stationary scenario between dark energy and dark matter. Due to the absence of a universally accepted interaction term introduced by a fundamental theory, the study is conducted over three different forms of chosen interaction terms. As an illustration three widely known models of $f(R, T)$ gravity are taken into consideration and used in the setup designed to study the problem. The study reveals that, the realization of the coincidence scenario is almost impossible for the popular models of $f(R, T)$ gravity, thus proving to be a major setback for these models.

Keywords: Dark energy, Dark matter, Modified gravity, coincidence, interaction.

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1 Introduction

Observational evidences from Ia supernovae, CMBR via WMAP, galaxy redshift surveys via SDSS indicated that the universe have entered a phase of accelerated expansion of late [1, 2, 3, 4, 5]. With this discovery the incompleteness of general relativity (GR) as a self sufficient theory of gravity came into foreground. Since no possible explanation of this phenomenon could be attributed inside the framework of Einstein's GR, a proper modification of the theory was required that will successfully incorporate the late cosmic acceleration. As the quest began, two different approaches regarding this modification came into light.

According to the first approach, cosmic acceleration can be phenomenally attributed to the presence of a mysterious negative energy component popularly known as *dark energy (DE)* [6]. Here we modify the right hand side of the Einstein's equation, i.e. in the matter sector of the universe. Latest observational data shows that the contribution of DE to the energy sector of the universe is $\Omega_d = 0.7$. With the passage of time, extensive search saw various candidates for DE appear in the scene. Some of the popular ones worth mentioning are Chaplygin gas models [7, 8], Quintessence Scalar field [9], Phantom energy field [10], etc. A basic feature of these models is that, they violate the strong energy condition i.e., $\rho + 3p < 0$, thus producing the observed cosmic acceleration. Recent reviews on DE can be found in [11, 12]

A different section of cosmologists resorted to an alternative approach for explaining the expansion. This concept is based on the modification of the gravity sector of GR, thus giving birth to modified gravity theories. A universe associated with a tiny cosmological constant, i.e. the Λ CDM model served as a prototype for this approach. It was seen that the model could satisfactorily explain the recent cosmic acceleration and passed a few solar system tests as well. But with detailed diagnosis it was revealed that the model was paralyzed with a few cosmological problems. Out of these, two major problems that crippled the model till date are the Fine tuning problem (FTP) and the Cosmic Coincidence problem (CCP). The FTP refers to the large discrepancy between the observed values and the theoretically predicted values of cosmological parameters. Numerous

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attempts to solve this problem can be found in the literature. Among them, the most impressive attempt was undertaken by Weinberg in [13]. Although the approaches for the solutions are different, yet, almost all of them are basically based on the fact that the cosmological constant may not assume an extremely small static value at all times during the evolution of the universe (as predicted by GR), but its nature should be rather dynamical [14]. These drawbacks reduced the effectiveness of the model, as well as its acceptability, and hence alternative modifications of gravity was sought for. Some of the popular models of modified gravity that came into existence in recent times are loop quantum gravity [15, 16], Brane gravity [17, 18, 19], $f(R)$ gravity [20, 21, 22], $f(T)$ gravity [23, 24, 25, 26], etc. Reviews on extended gravity theories can be found in [27, 28, 29].

In this work we will consider $f(R, T)$ model as the theory of gravity [30]. Over the years, several modifications to GR have been achieved, by generalizing the Einstein-Hilbert Lagrangian used in GR. $f(R)$ and $f(T)$ gravities are common examples of such modifications. In $f(R)$ gravity the Ricci scalar R is replaced by a general function of R in the Einstein-Hilbert action. Levi-Civita connection is used in the theory with only curvature for its formation. In $f(T)$ gravity, the torsion scalar T is replaced by a general function of T in the action of the teleparallel equivalent of general relativity (TEGR). Weitzenböck connection is used in the theory where the curvature is replaced by torsion. Both these theories can successfully explain the recent cosmic acceleration and passes several solar system tests [31, 32, 33, 34, 35].

$f(R, T)$ gravity is a novel attempt to unify the $f(R)$ and $f(T)$ gravities preserving the properties of the constituent theories. The evolution of the universe is explained under the combined effect of both curvature scalar, R and torsion, T . Over the past two years $f(R, T)$ gravity has evolved as a prospective and a very interesting version of modified gravity theory. Since the introduction of the theory, numerous works have been recorded in the literature, investigating its various aspects. The thermodynamic properties of the theory are studied in [36]. The energy conditions are studied in [37]. Anisotropic cosmology in the background of $f(R, T)$ gravity was studied in [38]. Cosmological solution via reconstruction program was studied in [39].

In [40] Bisabr studied cosmological coincidence problem in the background of $f(R)$ gravity. In [41] and [42], the problem has been studied in $f(G)$ and $f(T)$ gravities respectively. Motivated by these, we dedicate the present assignment to the study of the coincidence problem in $f(R, T)$ gravity. The paper is organized as follows: Basic equations of $f(R, T)$ gravity are furnished in section 2. In section 3, we discuss the coincidence problem. The set-up for the present study is discussed in section 4. We illustrate the designed set-up by a few examples in section 5, and finally the paper ends with a short conclusion in section 6.

2 Basic Equations of $f(R, T)$ Gravity

The action of the $f(R, T)$ gravity theory is given by

$$A = \int dx^4 \sqrt{-g} \left[\frac{f(R, T)}{16\pi G} + \mathcal{L} \right] \quad (1)$$

where \mathcal{L} determines the matter content of the universe. The energy momentum tensor of matter is defined as

$$T_{\mu\nu}^{(matter)} = -\frac{2}{\sqrt{-g}} \frac{\delta(\sqrt{-g}\mathcal{L})}{\delta g^{\mu\nu}} \quad (2)$$

We assume that the matter Lagrangian density depends only on the metric tensor components $g_{\mu\nu}$ so that

$$T_{\mu\nu}^{(matter)} = g_{\mu\nu} L_{(matter)} - \frac{2\delta L_{(matter)}}{\delta g^{\mu\nu}} \quad (3)$$

Taking the variation of the action with respect to the metric tensor, we get the field equations for the $f(R, T)$ gravity as follows,

$$R_{\mu\nu} f_R(R, T) - \frac{1}{2} g_{\mu\nu} f(R, T) + (g_{\mu\nu} \square - \nabla_\mu \nabla_\nu) f_R(R, T) = 8\pi G T_{\mu\nu}^{(matter)} - f_T(R, T) T_{\mu\nu}^{(matter)} - f_T(R, T) \Theta_{\mu\nu} \quad (4)$$

where ∇_μ is the covariant derivative associated with the Levi-Civita connection of the metric and $\square = \nabla_\mu \nabla^\mu$. Subscripts R and T represents derivative with respect to R and T respectively and $\Theta_{\mu\nu} = \frac{g^{\alpha\beta} \delta T_{\alpha\beta}}{\delta g^{\mu\nu}}$.

The energy-momentum tensor of the matter is given as

$$T_{\mu\nu}^{(matter)} = (\rho_m + p_m) u_\mu u_\nu + p_m g_{\mu\nu} \quad (5)$$

Using this the field eqns. (4) gives,

$$R_{\mu\nu}f_R(R, T) - \frac{1}{2}g_{\mu\nu}f(R, T) + (g_{\mu\nu}\square\nabla_\mu\nabla_\nu)f_R(R, T) = 8\pi GT_{\mu\nu}^{(matter)} + T_{\mu\nu}^{(matter)}f_T(R, T) + p_m g_{\mu\nu}f_T(R, T) \quad (6)$$

Here we will consider non-relativistic matter, i.e., cold dark matter and baryons ($p_m = 0$). So its obvious that the torsion contribution is totally coming from ordinary matter. An effective Einstein eqn. can be written from eqn.(6) as follows,

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi G_{eff}T_{\mu\nu}^{(matter)} + T_{\mu\nu}^{(d)} \quad (7)$$

where the effective gravitational matter dependant coupling in $f(R, T)$ gravity is given by,

$$G_{eff} = \frac{1}{f_R(R, T)} \left(G + \frac{f_T(R, T)}{8\pi} \right) \quad (8)$$

and the energy momentum tensor for dark energy is given by,

$$T_{\mu\nu}^{(d)} = \frac{1}{f_R(R, T)} \left[\frac{1}{2}g_{\mu\nu} (f(R, T) - Rf_R(R, T)) + (\nabla_\mu\nabla_\nu - g_{\mu\nu}\square)f_R(R, T) \right] \quad (9)$$

here prime denotes the non-equilibrium description of the field equations.

Now the metric describing the FRW universe is given by,

$$ds^2 = h_{\alpha\beta}dx^\alpha dx^\beta + \tilde{r}^2 d\Omega^2 \quad (10)$$

where $\tilde{r} = a(t)r$, $x^0 = t$, $x^1 = r$ and the metric $h_{\alpha\beta} = \text{diag}(-1, \frac{a^2}{1-kr^2})$. Obviously $a(t)$ is the time dependant scale factor, k is the scalar curvature and $d\Omega^2$ is the metric for the two-dimensional unit sphere. Now the field equations for the FRW universe can be given as follows,

$$3 \left(H^2 + \frac{k}{a^2} \right) = 8\pi G_{eff}\rho_m + \frac{1}{f_R} \left[\frac{1}{2} (Rf_R - f) - 3H (\dot{R}f_{RR} + \dot{T}f_{RT}) \right] \quad (11)$$

$$- \left(2\dot{H} + 3H^2 + \frac{k}{a^2} \right) = \frac{1}{f_R} \left[-\frac{1}{2} (Rf_R - f) + 2H (\dot{R}f_{RR} + \dot{T}f_{RT}) + \ddot{R}f_{RR} + \dot{R}^2 f_{RRR} + 2\dot{R}\dot{T}f_{RRT} + \ddot{T}f_{RT} + \dot{T}^2 f_{RTT} \right] \quad (12)$$

We re-write these eqns. as

$$3 \left(H^2 + \frac{k}{a^2} \right) = 8\pi G_{eff} (\rho_m + \rho_d) \quad (13)$$

$$- 2 \left(\dot{H} - \frac{k}{a^2} \right) = 8\pi G_{eff} (\rho_m + \rho_d + p_d) \quad (14)$$

where ρ_d and p_d are respectively the energy density and pressure of dark energy given by,

$$\rho_d = \frac{1}{8\pi GF} \left[\frac{1}{2} (Rf_R - f) - 3H (\dot{R}f_{RR} + \dot{T}f_{RT}) \right] \quad (15)$$

$$p_d = \frac{1}{8\pi GF} \left[-\frac{1}{2} (Rf_R - f) + 2H (\dot{R}f_{RR} + \dot{T}f_{RT}) + \ddot{R}f_{RR} + \dot{R}^2 f_{RRR} + 2\dot{R}\dot{T}f_{RRT} + \ddot{T}f_{RT} + \dot{T}^2 f_{RTT} \right] \quad (16)$$

Here $F = 1 + \frac{f_T(R, T)}{8\pi G}$.

The energy conservation equations for matter and dark sector are respectively given by,

$$\dot{\rho}_m + 3H\rho_m = Q \quad (17)$$

and

$$\dot{\rho}_d + 3H(1 + \omega_d)\rho_d = -Q \quad (18)$$

Here $\omega_d = \frac{p_d}{\rho_d}$ is the EoS parameter of the energy sector and Q is the interaction between the matter and the energy sector of the universe. The EoS parameter of the dark fluid is obtained as,

$$\omega_d = -1 + \frac{\ddot{R}f_{RR} + \dot{R}^2 f_{RRR} + 2\dot{R}\dot{T}f_{RRT} + \ddot{T}f_{RT} + \dot{T}^2 f_{RTT} - H (\dot{R}f_{RR} + \dot{T}f_{RT})}{\frac{1}{2} (Rf_R - f) - 3H (\dot{R}f_{RR} + \dot{T}f_{RT})} \quad (19)$$

3 The Coincidence problem

The cosmic coincidence problem has been a serious issue in recent times regarding various otherwise successful models of the universe. From the recent cosmological observations it is noted that the densities of the matter sector and the DE sector of the universe are almost identical in late times. This observation gives rise to a problem when we relate this to the fact that the matter and the energy component of the universe have evolved independently from different mass scales in the early universe. Then how do they reconcile to identical mass scales in the late universe! This is a major cosmological problem having its roots in the very formation of the models. Almost all the models of universe known till date more or less suffer from this phenomenon.

Numerous attempts to address the coincidence problem can be widely found in literature. Among them the most impressive are the ones which introduce a suitable interaction between the matter and the energy components of the universe, as given in the conservation equations (17) and (18). This approach makes use of the fact that the two sectors of the universe have not evolved independently from different mass scales, but have actually evolved together, interacting with each other, thus allowing a mutual flow of matter and energy between the two components. Due to this exchange, difference of densities, if any, gets diluted and a stationary scenario is witnessed in the present universe. Although the concept seems to be a really promising one, yet a problem persists. Till date there is no universally accepted interaction term, introduced by a fundamental theory. An attempt to address the coincidence problem in $f(R)$ gravity can be found in [40]. Similar attempts in $f(G)$ and $f(T)$ gravities can be found in [41, 42] respectively. A study of triple interacting DE model can be found in [43].

It is known that both dark energy and dark matter are not universally accepted facts, but concepts which are still at the speculation level. Due to this unknown nature of both dark energy and dark matter, it is not possible to derive an expression for the interaction term (Q) from the first principles. Such a situation, demands us to use our logical reasoning and propose various expressions for Q that will be reasonably acceptable. The late time dominating nature of dark energy indicates that Q must be considered a small and positive value. On the other hand a large negative value of interaction will make the universe dark energy dominated from the early times, thus leaving no scope for the condensation of galaxies. So the most logical choice for interaction should contain a product of energy density and the hubble parameter, because it is not only physically but also dimensionally justified. So $Q = Q(H\rho_m, H\rho_{de})$, where ρ_{de} is the dark energy density. Since here we are not planning to add any dark energy by hand, so the effective density resulting from the $f(R, T)$ gravity, ρ_d will replace ρ_{de} . This leads us to three basic forms of interactions as given below [45]:

$$b - model : Q = 3bH\rho_m \quad \eta - model : Q = 3\eta H\rho_d \quad \Gamma - model : Q = 3\Gamma H(\rho_m + \rho_d), \quad (20)$$

where b , η and Γ are the coupling parameters of the respective interaction models.

It is worth mentioning that due to its simplicity as well as viability, the most widely used interaction model is the b -model and is available widely in literature [44, 45, 46, 47].

4 The set-up

In this note we address the coincidence problem in $f(R, T)$ gravity. $f(R, T)$ gravity has evolved over the past few years as a candidate for modified gravity theory. From the literature it is known that $f(R, T)$ gravity is itself self competent in producing the late cosmic acceleration without resorting to any forms of dark energy. Therefore in order to keep it simple and reasonable, we do not consider any separate dark energy component by hand in the present study. The energy component evolving from the gravity theory itself is considered as the dark component responsible for the cosmic acceleration. The ratio of the densities of matter and dark energy is considered as, $r \equiv \rho_m/\rho_d$. Our prime objective is to devise a set-up that will take us close to a possible solution of the coincidence problem. We also want to set up a filtering process that will screen the favorable $f(R, T)$ models, that produce a stationary scenario of the component densities, r from the unfavorable ones which do not. The time evolution of r is as follows,

$$\dot{r} = \frac{\dot{\rho}_m}{\rho_d} - r \frac{\dot{\rho}_d}{\rho_d} \quad (21)$$

Using eqns. (17), (18) and (21), we obtain

$$\dot{r} = 3Hr\omega_d + \frac{Q}{\rho_d}(1+r) \quad (22)$$

Using the b-interaction given in eqn.(20), we get the expression for \dot{r} as,

$$\dot{r} = 3Hr (b + br + \omega_d) \quad (23)$$

where ω_d is given by eqn.(19). Now in order to comply with observations, it is required that universe should approach a stationary stage, where either r becomes a constant or evolves slower than the scale factor. In order to satisfy this $\dot{r} = 0$ in the present epoch, it leads to the following equation,

$$g_1(f, H, r_s, q) = 0 \quad (24)$$

where

$$g_1(f, H, r_s, q) = 3Hr_s \left[b + \frac{\dot{H}}{H^2} + q + br_s + \frac{1}{\frac{1}{2} \left(-f + Rf_R - 3H \left(\ddot{H} + 4H\dot{H} \right) f_{RR} - 12H\dot{H}f_{RT} \right)} \times \right. \\ \left. \left(6 \left(\ddot{H} + 4 \left(\dot{H}^2 + H\ddot{H} \right) \right) f_{RR} - 12 \left(\dot{H}^2 + H\ddot{H} \right) f_{RT} - H \left(6 \left(\ddot{H} + 4H\dot{H} \right) f_{RR} - 12H\dot{H}f_{RT} + 36 \left(\ddot{H} + 4H\dot{H} \right)^2 f_{RRR} \right. \right. \right. \\ \left. \left. \left. - 144H\dot{H} \left(\ddot{H} + 4H\dot{H} \right) f_{RRT} + 144\dot{H}^2 H^2 f_{RTT} \right) \right) \right] \quad (25)$$

and r_s is the value of r when it takes a stationary value.

Using the η -interaction given in eqn.(20), we get the expression for \dot{r} as,

$$\dot{r} = 3H [\eta + r (\eta + \omega_d)] \quad (26)$$

where ω_d is given by eqn.(19). In order to satisfy this $\dot{r} = 0$ in the present epoch, it leads to the following equation,

$$g_2(f, H, r_s, q) = 0 \quad (27)$$

where

$$g_2(f, H, r_s, q) = 3H \left[\eta + r_s \left(\frac{\dot{H}}{H^2} + q + \eta + \frac{1}{\left(\frac{1}{2} \left(-f + 6 \left(\dot{H} + 2H^2 \right) f_R - 3H \left(6 \left(\ddot{H} + 4H\dot{H} \right) f_{RR} - 12H\dot{H}f_{RT} \right) \right)} \right) \times \right. \right. \\ \left. \left(6 \left(\ddot{H} + 4 \left(\dot{H}^2 + H\ddot{H} \right) \right) f_{RR} - 12 \left(\dot{H}^2 + H\ddot{H} \right) f_{RT} - H \left(6 \left(\ddot{H} + 4H\dot{H} \right) f_{RR} - 12H\dot{H}f_{RT} \right) + 36 \left(\ddot{H} + 4H\dot{H} \right)^2 f_{RRR} \right. \right. \right. \\ \left. \left. \left. - 144H\dot{H} \left(\ddot{H} + 4H\dot{H} \right) f_{RRT} + 144\dot{H}^2 H^2 f_{RTT} \right) \right) \right] \quad (28)$$

Using the Γ -interaction given in eqn.(20), we get the expression for \dot{r} as,

$$\dot{r} = 3H [\Gamma r^2 + r (2\Gamma + \omega_d) + \Gamma] \quad (29)$$

where ω_d is given by eqn.(19). In this case, in order to satisfy $\dot{r} = 0$ in the present epoch, it leads to the following equation,

$$g_3(f, H, r_s, q) = 0 \quad (30)$$

where

$$g_3(f, H, r_s, q) = 3H \left[\Gamma + r_s^2 \Gamma + r_s \left(\frac{\dot{H}}{H^2} + q + 2\Gamma + \frac{1}{\left(\frac{1}{2} \left(-f + Rf_R \right) - 3H \left(6 \left(\ddot{H} + 4H\dot{H} \right) f_{RR} - 12H\dot{H}f_{RT} \right) \right)} \right) \times \right. \\ \left. \left(6 \left(\ddot{H} + 4 \left(\dot{H}^2 + H\ddot{H} \right) \right) f_{RR} - 12 \left(\dot{H}^2 + H\ddot{H} \right) f_{RT} - H \left(6 \left(\ddot{H} + 4H\dot{H} \right) f_{RR} - 12H\dot{H}f_{RT} + 36 \left(\ddot{H} + 4H\dot{H} \right)^2 f_{RRR} \right. \right. \right. \right. \\ \left. \left. \left. - 144H\dot{H} \left(\ddot{H} + 4H\dot{H} \right) f_{RRT} + 144\dot{H}^2 H^2 f_{RTT} \right) \right) \right] \quad (30)$$

$$-144H\dot{H}\left(\ddot{H}+4H\dot{H}\right)f_{RRT}+144\dot{H}^2H^2f_{RTT}\Big)\Big)\Big)\Big]\quad (31)$$

In our analysis we will consider H_0 , r_0 and q_0 as the present day values of H , r and q respectively. As far as q is concerned, we start from the best fit parametrization obtained directly from observational data. Here we use a two parameter reconstruction function for $q(z)$ [48, 49]

$$q(z) = \frac{1}{2} + \frac{q_1 z + q_2}{(1+z)^2} \quad (32)$$

On fitting this model to Gold data set, we get $q_1 = 1.47^{+1.89}_{-1.82}$ and $q_2 = -1.46 \pm 0.43$ [49]. We consider $z_0 = 0.25$ and using these values in eqn.(32), we get $q_0 \approx -0.2$. From recent observations, we obtain $r_0 \equiv \frac{\rho_m(z_0)}{\rho_T(z_0)} \approx \frac{3}{7}$ [50, 51, 52]. The present value of Hubble parameter, H_0 is taken as 72, in accordance with the latest observational data.

5 Illustration

Here we consider the scale-factor, of the Λ CDM universe, which describes the late universe quite satisfactorily. For a DE component with EoS w , we have $\rho(t) \sim a^{-3(1+w)}$ and thus $a(t) \sim t^{2/(3(1+w))}$. So in the early stage, where there is only radiation ($w = 1/3$) then $a(t) \sim t^{1/2}$. In the late universe, when we have only matter $a(t) \sim t^{2/3}$. Therefore we consider our scale factor as,

$$a = a_0 t^{2/3} \quad (33)$$

where a_0 is a constant.

In order to illustrate the above set-up numerically we consider three different $f(R, T)$ gravity models found in literature and test them for the coincidence phenomenon. These models are used because they pass most of the cosmological and solar system tests. The three models are [56]

Model1:

$$f(R, T) = \mu R^{n_1} + \nu T^m \quad (34)$$

where μ , n_1 , ν and m are constants.

Model2:

$$f(R, T) = R^p (\text{Log}(\alpha R))^{q_3} + \sqrt{-T} \quad (35)$$

where p , $\alpha > 0$ and $q_3 \neq 0$ are constants.

Model3:

$$f(R, T) = R + \beta R^{-n_2} + \sqrt{-T} \quad (36)$$

where $n_2 \neq 0$ and β are constants.

Using the model 1, i.e., eqn.(34) and eqn. (33) in eqn. (25), we get the following expression for the dynamical quantity g_1 ,

$$\begin{aligned} g_1^{\text{model1}} = & \frac{1}{t} \left[3nr \left(b - \frac{1}{n} + q + br + \left\{ -\frac{2^{-1+n_1} 3^{n_1} n (-1+n_1) n_1 \left(\frac{2n}{t^3} - \frac{4n^2}{t^3} \right) \left(-\frac{n}{t^2} + \frac{2n^2}{t^2} \right)^{-2+n_1} \mu}{t} \right. \right. \right. \\ & \left. \left. \left. + \frac{1}{2} \left(-6^{n_1} \left(-\frac{n}{t^2} + \frac{2n^2}{t^2} \right)^{n_1} \mu + 6^{n_1} n_1 \left(-\frac{n}{t^2} + \frac{2n^2}{t^2} \right)^{n_1} \mu - 6^m \left(-\frac{n^2}{t^2} \right)^m \nu \right) \right\}^{-1} \times \right. \\ & \left. \left(6^{-1+n_1} (-2+n_1) (-1+n_1) n_1 \left(\frac{2n}{t^3} - \frac{4n^2}{t^3} \right)^2 \left(-\frac{n}{t^2} + \frac{2n^2}{t^2} \right)^{-3+n_1} \mu + 6^{-1+n_1} (-1+n_1) n_1 \right. \right. \end{aligned}$$

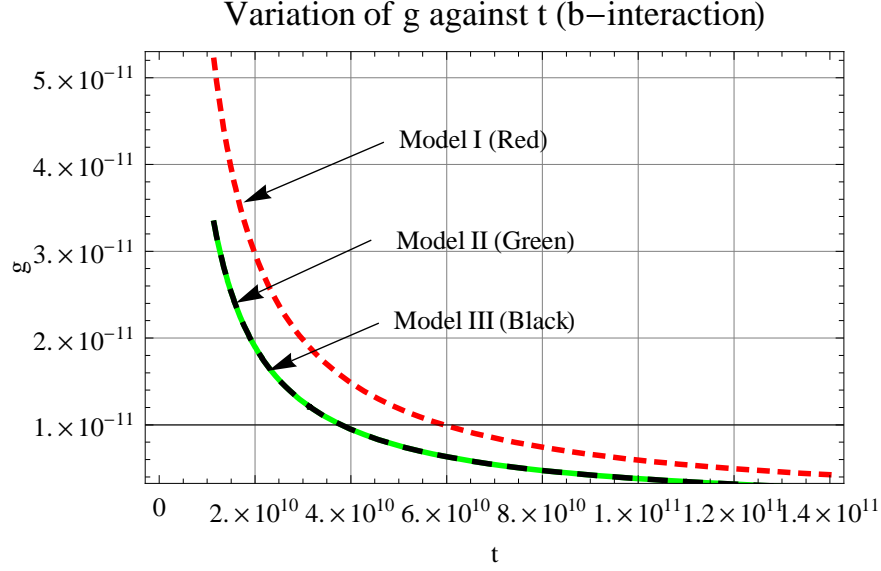


Fig.1

Fig 1 : The plot of $g(f_0, H_0, r_{s0}, q_0)$ against t for model1 (red), model2 (green) and model3 (black) using b interaction. The other parameters are considered as $q = -0.2, r = 3/7, b = 1.5, \mu = 0.01, \nu = 0.05, n_1 = 0.5, m = 3, \alpha = 5.7 \times 10^{-61}, p = 1, q_3 = -1, n_2 = -0.9, \beta = 0.646 \times 10^{-4}$

$$\left(-\frac{6n}{t^4} + \frac{12n^2}{t^4} \right) \left(-\frac{n}{t^2} + \frac{2n^2}{t^2} \right)^{-2+n_1} \mu - \frac{6^{-1+n_1} n (-1+n_1) n_1 \left(\frac{2n}{t^3} - \frac{4n^2}{t^3} \right) \left(-\frac{n}{t^2} + \frac{2n^2}{t^2} \right)^{-2+n_1} \mu}{t} \right) \right] \quad (37)$$

Similarly expressions for g_1 is obtained for the other two $f(R, T)$ models. Expressions for g_2 and g_3 are also found for all the three gravity models. As it can be seen from above that the expressions are really lengthy, so we do not include all of them in the manuscript.

We have generated plots for g_1, g_2 and g_3 against cosmic time, t for each of the three models in the figures 1, 2 and 3, for all the three forms of interactions, b, η and Γ respectively. Particular numerical values for the involved parameters have been considered which are in accordance with the recent observational data [53, 54, 55, 56, 57].

6 Discussion and Conclusion

From the figures it is evident that the stationary scenario is not realized for all the three models when the cosmic time corresponds to the age of the universe, i.e. 14×10^9 years. It is seen that near the zero line the trajectories assume asymptotic nature. Thus there is no realistic possibility of the trajectory to intersect the zero axis, producing a non-stationary scenario. The asymptotic nature of the curves are indicative of the fact that as time evolves the trajectories move closer and closer to the zero mark thus alleviating the coincidence problem substantially, but never produces a satisfactory solution. This reveals a basic flaw in the framework of the $f(R, T)$ models which are otherwise considered to be quite consistent with the solar system tests. In the figures 1, 2 and 3 the trajectories have been generated for b, η and Γ -interaction respectively. In all the three cases, the trajectories for model 2 and 3 coincide with each other. The trajectory for model 1 is distinct from the other models in case of b -interaction. But in case of η and Γ interactions even the trajectory for model 1 move closer to the other trajectories almost coinciding with them.

In [41, 42] coincidence problem has been addressed in $f(G)$ and $f(T)$ gravity models respectively. In both these assignments the b -interaction has been identified as the most suitable form of interaction describing the

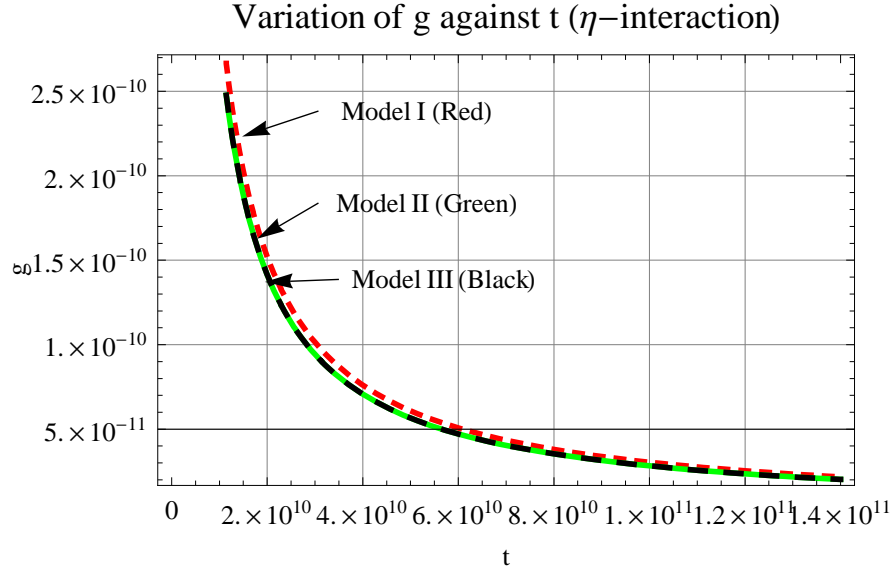


Fig.2

Fig 2 : The plot of $g(f_0, H_0, r_{s0}, q_0)$ against t for model1 (red), model2 (green) and model3 (black) using η interaction. The other parameters are considered as $q = -0.2, r = 3/7, b = 1.5, \mu = 0.01, \nu = 0.05, n_1 = 0.5, m = 3, \alpha = 5.7 \times 10^{-61}, p = 1, q_3 = -1, n_2 = -0.9, \beta = 0.646 \times 10^{-4}$

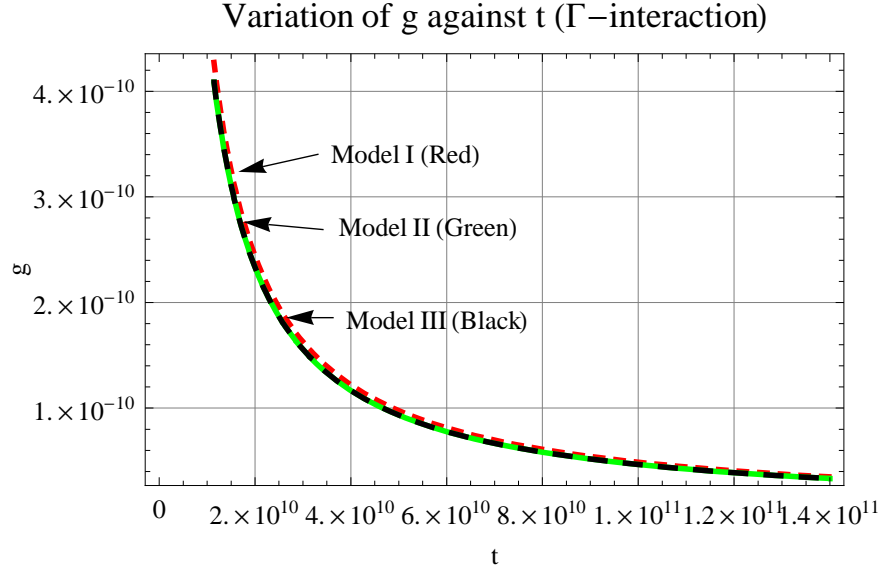


Fig.3

Fig 3 : The plot of $g(f_0, H_0, r_{s0}, q_0)$ against t for model1 (red), model2 (green) and model3 (black) using Γ interaction. The other parameters are considered as $q = -0.2, r = 3/7, b = 1.5, \mu = 0.01, \nu = 0.05, n_1 = 0.5, m = 3, \alpha = 5.7 \times 10^{-61}, p = 1, q_3 = -1, n_2 = -0.9, \beta = 0.646 \times 10^{-4}$

late universe. But in the present study, there is no such reason to consider the above mentioned fact. But it should be mentioned that the present study resembles the study in [41] in the fact that in both the papers coincidence scenario is not realized for the well-known models of the respective gravity theories. Finally it must be stated that from the set-up that we have designed in this assignment, we can generate as well as filter various models of $f(R, T)$ gravity which are completely free from the coincidence problem. For the time being we keep it as a future assignment.

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